6. PARTIAL FRACTIONS

Quick Review

1. An expression of the form $f(x) = a_0 + a_1x + a_2x^2 + \ldots + a_nx^n$, where n is a non negative integer and $a_0, a_1, a_2, \ldots, a_n$ are real numbers such that $a_n \neq 0$, is called a polynomial in x of degree n.
2. **Division Algorithm :** If $f(x)$, $0 \neq g(x)$ are two polynomials, then $\exists$ polynomials $q(x)$, $r(x)$ uniquely such that $f(x) = q(x)\,g(x) + r(x)$ where $r(x) = 0$ or $\deg r(x) < \deg g(x)$. The polynomial $q(x)$ is called quotient and the polynomial $r(x)$ is called remainder of $f(x)$ when divided by $g(x)$.
3. If $f(x)$, $g(x)$ are two polynomials and $g(x) \neq 0$ then $\frac{f(x)}{g(x)}$ is called a rational function or rational fraction.
4. A fraction $\frac{f(x)}{g(x)}$ is said to be a proper fraction if $\deg f(x) < \deg g(x)$. Otherwise it is said to be an improper fraction.
5. If a rational function can be expressed as a sum of two or more proper fractions, then each fraction is called a partial fraction of the given function.
6. Let $\frac{f(x)}{g(x)}$ be a proper fraction.

   (i) If $(ax + b)^n$ where $n \in N$, is a factor of $g(x)$ then the partial fractions corresponding to this factor are $\frac{A_1}{ax+b} + \frac{A_2}{(ax+b)^2} + \ldots + \frac{A_n}{(ax+b)^n}$, where $A_1, A_2, \ldots A_n$ are constants.

   (ii) If $(ax^2 + bx + c)^n$ where $n \in N$, is a factor of $g(x)$ then the partial fractions corresponding to this factor are $\frac{A_1x+B_1}{ax^2+bx+c} + \frac{A_2x+B_2}{(ax^2+bx+c)^2} + \ldots + \frac{A_nx+B_n}{(ax^2+bx+c)^n}$ where $A_1, A_2, \ldots A_n, B_1, B_2, \ldots, B_n$ are constants.
7. $\frac{px+q}{(x-a)(x-b)} = \frac{pa+q}{(x-a)(a-b)} + \frac{pb+q}{(b-a)(x-b)}$.
8. $\frac{px+q}{(x-a)(x-b)(x-c)} = \frac{pa+q}{(x-a)(a-b)(a-c)} + \frac{pb+q}{(b-a)(x-b)(b-c)} + \frac{pc+q}{(c-a)(c-b)(x-c)}$.
9. $\frac{px+q}{x^2(x-a)} = \frac{-q}{ax^2} - \frac{pa+q}{a^2x} + \frac{pa+q}{a^2(x-a)}$.
10. If $|x| < a$, $|x| < b$ then the coefficient of x^n in $\frac{x}{(x-a)(x-b)}$ is $\frac{a^n - b^n}{a-b} \cdot \frac{1}{a^nb^n}$.